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Locating Replenishment Stations for Electric Vehicles: Application to Danish Traffic Data

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Abstract

Environment-friendly electric vehicles have gained substantial attention in governments, industry and universities. The deployment of a network of recharging stations is essential given their limited travel range. This paper considers the problem of locating electronic replenishment stations for electric vehicles on a traffic network with flow-based demand. The objective is to optimize the network performance, for example to maximize the flow covered by a prefixed number of stations, or to minimize the number of stations needed to cover traffic flows. Two mixed integer linear programming formulations are proposed to model the problem. These models are tested on real-life traffic data collected in Denmark. Computational results are presented.

1 Introduction

During the past few decades, environmental concerns have generated a renewed interest in electrical vehicles. The current yearly worldwide sales of fully electric vehicles now stand at around 20,000 units. The market is expected to grow to 750,000 units in 2020 in the European Union (EU) alone (Philippe (2011)). The

major advantage of electrical vehicles is that they are more environment-friendly than traditional vehicles. They emit much less CO₂ and almost no air pollutants. However, electrical vehicles have a limited driving range due to the low density of their batteries. It is therefore important to supply electrical vehicles with recharging stations to lengthen their autonomy. Around 1000 electric charging stations are now in operation in the USA, distributed in 39 states (U.S. Department of Energy (2011)). It is predicted that the number of public charging points in the EU will grow to close to two million by 2017 (Pike Research (2011)). In Europe, the approximate investment in such infrastructure over the next seven years is likely to be about five billion Euros (Pike Research (2011)). This expenditure will largely be motivated by local government initiatives aimed at boosting the expansion of public charging infrastructures for electric vehicles. It is estimated that more than 4.7 million electric vehicle charge points will be installed worldwide by 2015 (Pike Research (2011)).

Locating stations is a central issue in the deployment of charging infrastructure. In this work, we investigate the problem of locating electronic replenishment stations for electric vehicles on the basis of traffic flows. Two models will be developed and compared. In the first one, the aim is to maximize the total flow captured by a given number of stations. In the second model the aim is to minimize the number of stations required to capture the entire flow. In both cases we impose a driving range constraint which ensures that every stretch of at least r km is covered by at least one station. The problem belongs to the family of Flow Interception Facility Location Problems (FIFLPs) in which demand is expressed by origin-destination (OD) flows on a directed network. It is assumed that drivers can always replenish at their origin or at their destination. This means that replenishment is not an issue on OD paths not exceeding r km.

The FIFLP was introduced by Hodgson (1981) in relation with the location of daycare facilities. Over the past 30 years, different models have been proposed to characterize a wide range of applications, such as the location of police inspection stations (Hodgson et al. (1996), Gendreau et al. (2000) and Selmić et al. (2010)), of road detecting sensors (Liu and Danczyk (2009)), of gas and refueling stations (Kuby et al. (2009), Lim and Kuby (2010), Upchurch et al. (2009), Wang and Lin (2009) and Wang and Wang (2010)), and so on. Boccia et al. (2009) provide a review on the problems, models and methods for FIFLPs.

The main contribution of this paper is the development of two models for the Electronic Replenishment Station Location Problem (ERSLP) and their assessment on real-life traffic data collected in Denmark. The remainder of the paper is organized as follows. A formal problem description and two mathematical models

are provided in Section 2. The data and a data preprocessing algorithm and the computational results are presented in Section 3, followed by conclusions in Section 4.

2 Formal problem description and mathematical models

A natural way to model the ERSLP is to use a flow capturing formulation. However, as rightly noted by Lim and Kuby (2010), the ERSLP is not a standard FIFPL because several facilities may be required to cover a flow. Several formulations are possible, depending on how the variables and constraints are defined. Lim and Kuby (2010) have developed a model which requires the enumeration of all facility combinations. Here we develop a more parsimonious but equivalent model applicable to relatively large scale applications. Our model works with a directed graph $G = (V, A)$, where V is a set of nodes and $A = \{(i, j) : i, j \in V, i \neq j\}$ is a set of arcs. A node can represent the origin or the destination of an OD path or potential location sites for stations. The set of OD paths is denoted by P . Let f_p be the traffic flow of path p . Each path $p \in P$ is represented by an ordered sequence of nodes $V^p = (v_1^p, \dots, v_l^p)$. Different paths can share some nodes. The set of all nodes is $V = V^1 \cup \dots \cup V^{|P|}$. We denote by t_{ij} the length of a shortest path from i to j . The driving range of a vehicle is the maximal distance it can drive without replenishing and is denoted by r . We denote by S_{ip} the maximal ordered subset $\{i, \dots, h\}$ of a subpath p , starting at node i , that can be driven without replenishment, i.e., such that $t_{ih} \leq r$.

2.1 Maximal flow capture model

In the maximal flow capture model (MFCM), the aim is to maximize the captured flow using m stations, subject to the driving range constraint. This model uses binary variables y_j equal to 1 if and only if a station is located at node j , binary variables z_p equal to 1 if and only if the flow of OD path p is covered by sufficient number of stations. To reduce the size of the models, we only consider those OD paths whose length exceeds r and undominated paths: path p dominates path p' if $V^{p'} \subseteq V^p$. Let P' be the resulting set of paths. The MFCM is as follows:

MFCM:

$$\text{maximize } \sum_{p \in P'} f_p z_p \quad (1)$$

$$\text{such that } \sum_{j \in V} y_j = m \quad (2)$$

$$\sum_{j \in S_{ip}} y_j \geq z_p \quad \forall p \in P', i \in V^p \quad (3)$$

$$y_j \in \{0, 1\} \quad \forall j \in V \quad (4)$$

$$z_p \in \{0, 1\} \quad \forall p \in P'. \quad (5)$$

In this formulation, the objective maximizes the traffic covered by the stations. Constraints (2) impose the location of m stations. They ensure that no feasible solution is excluded since all locations in V are considered as candidates. Constraints (3) state that every subpath of an OD path p that is covered (i.e. $z_p = 1$) contains a replenishment station within r units of each of its nodes. Constraints (4) and (5) define the binary variables.

2.2 Total flow capture model

In the total flow capture model (TFCM), the aim is to cover all traffic with the least number of stations. The model is

TFCM:

$$\text{minimize } \sum_{j \in V} y_j \quad (6)$$

$$\text{such that } \sum_{j \in S_{ip}} y_j \geq 1 \quad \forall p \in P', i \in V^p \quad (7)$$

$$y_j \in \{0, 1\} \quad \forall j \in V. \quad (8)$$

In this model, constraints (7) ensure that each OD path is covered by at least one station and that the driving range constraints are satisfied.

3 Computational results

Solving MFCM or TFCM requires traffic data. In Denmark which serves as a basis for this study, such data are provided by the government which conducts traffic surveys every 10 years. These surveys include Daily Trip Schedules which describe

Number of paths	Length interval
186	0 km – 49 km
354	50 km – 99 km
667	100 km – 159 km
330	160 km – 199 km
421	200 km – 299 km
254	300 km – 499 km
57	500 km – 1210 km

Table 1: Path length distribution

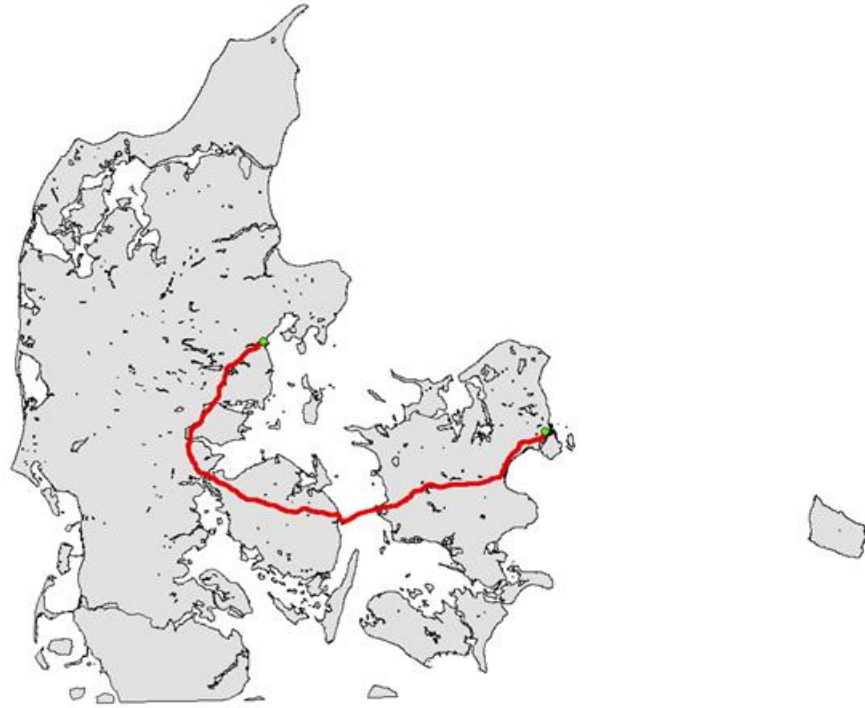
Number of nodes	Degree interval
50	300 – 530
171	200 – 299
229	150 – 199
647	100 – 149
278	80 – 90
1443	50 – 80
7407	20 – 49
11,387	10 – 19
84,246	1 – 9

Table 2: Node degree distribution

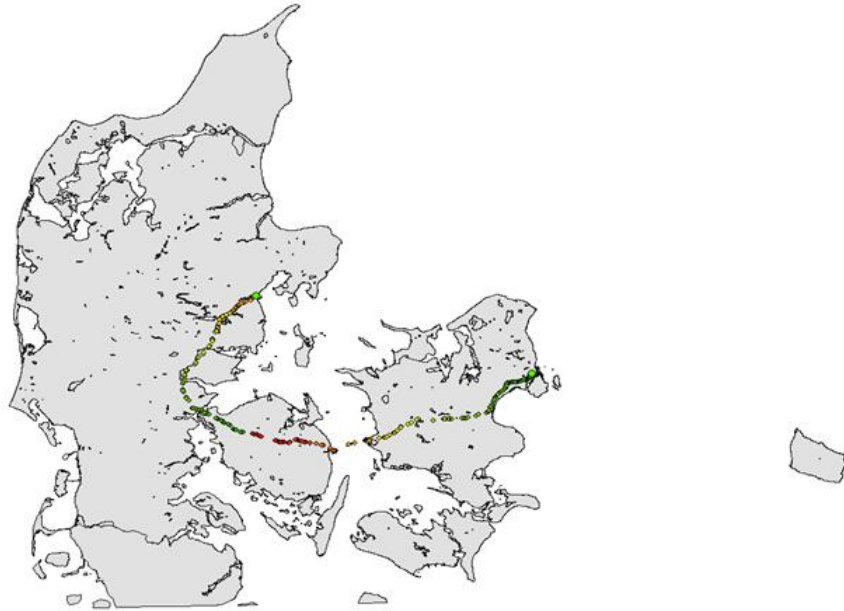
the OD paths of each interviewed person for a given day. These paths are then transformed into shortest path by means of an algorithm based on a route choice model. Every shortest path is represented by a sequence of nodes including junctions, highway entry and exit points, and other potential locations for stations. Figure 1 depicts a shortest path from Copenhagen to Aarhus.

In the traffic data, there are 2269 paths of varying lengths and 105,827 nodes. Table 1 gives the distribution of path lengths of paths and Table 2 gives the distribution of node degrees. The degree of node represents the number of paths covering it. The average node degree is 8.7.

To reduce the instance size, we preprocess the data by removing nodes that are only covered by a relatively small number of paths and that are located close to other nodes. Two parameters are used to control the degree of reduction in the instance size. One is called “LBDegree”: only those nodes with a degree less than LBDegree will be considered to be removed from the paths. The other parameter is called “LBDist”. We only remove a node from a path if the distances between its predecessor and its successor is shorter than LBDist. This is to prevent removing all nodes from a path. The preprocessing algorithm is given in Algorithm 1. It is implemented in C#. Table 3 gives the number of nodes in the reduced instance



(a) Shortest path from Copenhagen to Aarhus.



(b) Nodes in the shortest path from Copenhagen to Aarhus.

Figure 1: Shortest path and potential station locations for a given OD pair between Copenhagen and Aarhus

Algorithm 1 Node removal algorithm.

```
1: Sort the nodes in non-decreasing order of their degrees;
2: for all  $v \in V$  do
3:   if The degree of  $v$  is less than  $LBDegree$  then
4:     for all  $p \in P$  do
5:       if  $p$  covers  $v$  and the distance between  $v$ 's predecessor and successor is
         less than  $LBDist$  then
6:         remove  $v$  from path  $p$ ;
7:       end if
8:     end for
9:   end if
10: end for
```

$LBDegree \backslash LBDist(m)$	5000	10000	20000	30000
0	105,827	105,827	105,827	105,827
3	58,001	57,276	57,136	57,124
5	41,270	39,996	39,646	39,603
10	24,725	22,704	21,893	21,705
50	8937	5865	4315	3734
80	7941	4786	3192	2547
100	7808	4644	3022	2368
150	7394	4209	2567	1887
1000	7219	3990	2313	1615

Table 3: Number of nodes in the reduced instances for different parameter settings of ($LBDegree$, $LBDist$).

for different values of ($LBDegree$, $LBDist$). With $LBDist = 5000m$, by increasing $LBDegree$ from 0 to 50, the instance size is reduced by more than 90%, but it cannot be further reduced significantly when $LBDegree$ is greater than 50. However, increasing $LBDist$ can still reduce instance size when $LBDegree > 50$. When $LBDegree = 1000$, which means that any node can be removed as long as the $LBDist$ requirement is satisfied, the instance size can still be reduced by 75% by increasing $LBDist$ from 5km to 30km.

3.1 Results of the maximal flow capture model

The MFCM model was tested with different values of m (5, 10, 50, 80 and 100) on the reduced instances generated by different parameter settings ($LBDegree$, $LBDist$). The results are provided in Table 4. For each reduced instance, the first row shows the number of paths covered when m is 5, 10, 50, 80 and 100, respectively, and the second row shows the corresponding percentage. The last row gives the running time. For example, for the reduced instance generated by the parameter setting

(150, 20,000), there are 469, 661, 1009, 1060 and 1062 paths covered by the selected 5, 10, 50, 80 and 100 stations, respectively. These paths correspond to 44, 62, 95, 99 and 100 percent of all the paths. The corresponding running times are 70, 349, 201, 82,864 and 6 seconds. Figure 2 illustrates the locations of stations on the map of Denmark for the reduced instance generated by the parameter setting (150, 20,000). The lines on the map are roads with major traffic flows in Denmark.

In general, the more the instance size is reduced, the shorter the running time is and the worse the solution quality is. Comparing the reduced instance generated by the parameter setting (150, 10,000) with the problem by the parameter setting (1000, 30,000), we can see that the instance size is reduced by 62%, the running time is shortened by more than 83% and the solution deteriorates by less than 9%. From the table, the parameter setting (150, 30,000) seems to provide a good trade-off between computational time and solution quality.

3.2 Results of the total flow capture model

In the TFCM, we have set the driving range to 160 km. The model was run on different reduced instances. The results are shown in Table 5. For each test, the minimum number of stations needed, the size of the model and the running time are given. For example, for the reduced instance generated by the parameter setting (150, 10,000), at least 68 stations are needed. There are 4209 variables and 46,906 constraints in the model, and it takes 4987 seconds to solve it. The locations of the 68 stations are depicted in Figure 3. In general, the more the instance size is reduced, the larger is the number of stations needed.

4 Conclusions

The deployment of charging infrastructure is important for electric vehicles due to their short travel range. We have considered the problem of locating electrical replenishment stations in Denmark based on data collected over the past ten years. To meet different criteria, we have presented two models. In the first model, the coverage of paths is maximized given a fixed number of stations. In the second model, the travel range is considered explicitly and the objective is to minimize the number of stations needed to supply enough endurance to the vehicles on long trips.

We have preprocessed the data to reduce the instance size and thus decrease computation time. Two parameters were used to control the aggregation level. Solutions to the reduced instances of different sizes were presented. It was found

$LBDegree \setminus LBDist$		10,000	20,000	30,000
150	$\sum_{p \in P'} z_p$	(469, 688, 1035, 1062, 1062)	(469, 661, 1009, 1060, 1062)	(469, 647, 990, 1049, 1062)
	$(\sum_{p \in P'} z_p) / P' $	(44, 65, 97, 100, 100)%	(44, 62, 95, 99, 100)%	(44, 61, 93, 99, 100)%
	Running time (s)	(150, 767, 94687, 6, 6)	(70, 349, 201, 82864, 6)	(45, 417, 167, 64, 9)
1000	z_p	(378, 611, 1027, 1062, 1062)	(404, 599, 983, 1027, 1062)	(406, 573, 946, 1027, 1055)
	$(\sum_{p \in P'} z_p) / P' $	(36, 58, 97, 100, 100)%	(38, 56, 93, 97, 100)%	(38, 54, 89, 97, 99)%
	Running time (s)	(42, 155, 4945, 1, 1)	(2, 2, 14, 1, 1)	(1, 1, 1, 1, 1)

Table 4: Results of the maximal flow capture model for different values of m on the reduced instances generated by different parameter settings of $(LBDegree, LBDist)$.



Figure 2: The locations of the stations for $m = 5, 10, 50, 80$ and 100 in the solution to the reduced instance generated by the parameter setting (LBDegree = 150, LBDist = 20,000).

$LBDegree \backslash LBDist$	10,000	20,000	30,000
150	68	82	93
	4209 variables 46,906 constraints	2566 variables 42,449 constraints	1887 variables 41,055 constraints
	time = 4987s	time = 640s	time = 7s
1000	70	90	107
	3990 variables 16,264 constraints	2313 variables 8274 constraints	1615 variables 5521 constraints
	time = 1700s	time = 2.17s	time = 0.3s

Table 5: Results of the total flow capture model on different reduced instances generated by different parameter settings of $(LBDegree, LBDist)$.

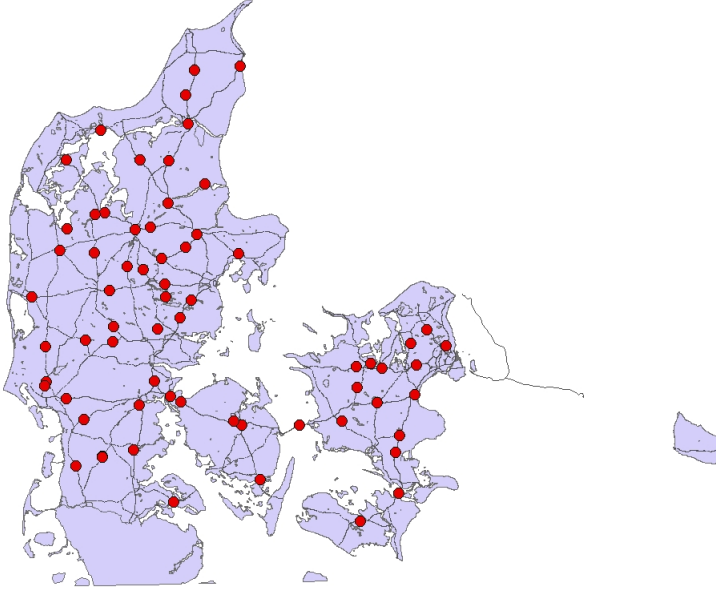


Figure 3: Locations of the 68 stations in the solution to the reduced instance generated with the parameter setting $(LBDegree = 150, LBDist = 10,000)$.

that the more the instance size is reduced, the larger is the number of stations needed and 68 stations are sufficient to ensure the driving range for the long paths.

In the future, more accurate models consisting of more real-life constraints and objectives can be investigated. Fast metaheuristic, e.g., adaptive large neighbourhood search, can also be developed to solve large scale problems.

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References

- Boccia, M., Sforza, A., and Sterle, C. (2009). Flow intercepting facility location: Problems, models and heuristics. *Journal of Mathematical Modelling and Algorithms*, 8(1):35–79.
- Gendreau, M., Laporte, G., and Parent, I. (2000). Heuristics for the location of inspection stations on a network. *Naval Research Logistics*, 47(4):287–303.
- Hodgson, M. J. (1981). The location of public facilities intermediate to the journey to work. *European Journal of Operational Research*, 6(2):199–204.
- Hodgson, M. J., Rosing, K. E., and Zhang, J. J. (1996). Locating vehicle inspection stations to protect a transportation network. *Geographical Analysis*, 28(4):299–314.
- Kuby, M., Lines, L., Schultz, R., Xie, Z., Kim, J.-G., and Lim, S. (2009). Optimization of hydrogen stations in Florida using the Flow-Refueling Location Model. *International Journal of Hydrogen Energy*, 34(15):6045–6064.
- Lim, S. and Kuby, M. (2010). Heuristic algorithms for siting alternative-fuel stations using the Flow-Refueling Location Model. *European Journal of Operational Research*, 204(1):51–61.
- Liu, H. X. and Danczyk, A. (2009). Optimal sensor location for freeway bottleneck identification. *Computer-Aided Civil and Infrastructure Engineering*, 24(8):535–550.

- Philippe, J. (2011). European policy on electric vehicles. Conference on Electric vehicles - Challenges of the New Mobility, Sofia.
- Pike Research (2011). Strategic technology and market analysis of electric vehicle charging infrastructure in Europe. <http://www.reportlinker.com/p0503640/Strategic-Technology-and-Market-Analysis-of-Electric-Vehicle-Charging-Infrastructure-in-Europe.pdf>.
- Selmić, M., Teodorović, D., and Vukadinović, K. (2010). Locating inspection facilities in traffic networks: an artificial intelligence approach. *Transportation Planning and Technology*, 33(6):481–493.
- Upchurch, C., Kuby, M., and Lim, S. (2009). A model for location of capacitated alternative-fuel stations. *Geographical Analysis*, 41(1):85–106.
- U.S. Department of Energy (2011). Alternative fueling station total counts by state and fuel type. http://www.afdc.energy.gov/afdc/fuels/stations_counts.html.
- Wang, Y.-W. and Lin, C.-C. (2009). Locating road-vehicle refueling stations. *Transportation Research Part E - Logistics and Transportation Review*, 45(5):821–829.
- Wang, Y.-W. and Wang, C.-R. (2010). Locating passenger vehicle refueling stations. *Transportation Research Part E - Logistics and Transportation Review*, 46(5):791–801.